Masses and internal structure of mesons in the string quark model

L.D. Soloviev
Department of Physics, University of Michigan, Ann Arbor, MI 48109-1120, USA,
and
Institute for High Energy Physics, 142284, Protvino, Russia
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The relativistic quantum string quark model, proposed earlier, is applied to all mesons, from pion to Υ, lying on the leading Regge trajectories (i.e., to the lowest radial excitations in terms of the potential quark models). The model describes the meson mass spectrum, and comparison with measured meson masses allows one to determine the parameters of the model: current quark masses, universal string tension, and phenomenological constants describing nonstring short-range interaction. The meson Regge trajectories are in general nonlinear; practically linear are only trajectories for light-quark mesons with non-zero lowest spins. The model predicts masses of many new higher-spin mesons. A new $K^*(1^-)$ meson is predicted with mass 1910 Mev. In some cases the masses of new low-spin mesons are predicted by extrapolation of the phenomenological short-range parameters in the quark masses. In this way the model predicts the mass of $\eta_b(1S)(0^{-+})$ to be 9500 ± 30 MeV, and the mass of $B_c(0^-)$ to be 6400 ± 30 MeV (the potential model predictions are 100 Mev lower). The relativistic wave functions of the composite mesons allow one to calculate the energy and spin structure of mesons. The average quark-spin projections in polarized ρ-meson are twice as small as the nonrelativistic quark model predictions. The spin structure of K^* reveals an 80% violation of the flavour $SU(3)$. These results may be relevant to understanding the "spin crises" for nucleons.

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I. INTRODUCTION

The naive quark model of hadrons, while attractive in its simplicity, is not quite so simple under closer examination. It is not relativistic since it contains a confinement potential proportional to a space distance. One can introduce a quasipotential dependent on distance and momentum, which makes the wave equation Lorentz covariant, but then the phenomenological quasipotential is not simple. The model contains constituent quarks which are purely phenomenological notions. Their masses are not fundamental and can vary from mesons to hadrons and even from one meson to another. Their spins are not fundamental either; and the "spin crises" for nucleons suggest that the quark spins should be different from 1/2, or that the naive quark model is too naive.

In Refs. [1,2] an alternative, string quark model (SQM) has been proposed which contains neither potential nor constituent quarks. The physical origin of confinement and constituent quarks, the gluon field, is taken into account explicitly, in an approximation of the quantum Nambu-Goto string. The string provides a confinement mechanism and, since the string is a physical object with its own energy-momentum and angular momentum, the quarks at the ends of the string are fundamental quarks with current masses and spin 1/2.

The application of SQM in Ref. [1] was confined to a particular type of leading meson Regge trajectories. Here we consider all four types of them, obtain relativistic wave functions of composite mesons, and calculate the internal (energy and spin) structure of mesons.

As in Ref. [1], we consider only the simplest string configuration — the rotating straight line, which is responsible for the leading Regge trajectories of mesons. The daughter trajectories (i.e., the higher radial excitations in terms of potential models) correspond to vibrations of the string.

The model is quantized in accord with Poincaré invariance and, due to account of quark spins, contains no tachyons.

The model predicts that the Regge trajectories for light-quark mesons with lowest spin 1 (ρ-type and b_1-type) are practically linear. The corresponding trajectories for heavy-light-quark mesons are not linear, but, to a good approximation, can be represented by straight lines for spins less than 6 by replacing the argument m^2 by $(m - m_h)^2$, where m_h is the heavy-quark mass. The slopes of these straight lines are bigger than for the light-quark mesons, and increase with m_h, the limit value being twice as big as for the light-quark mesons. The trajectories for heavy quarkonia are essentially nonlinear.

The Regge trajectories with lowest spin 0 (π-type and a_0-type) are always nonlinear in the low-spin region.

The model describes masses of all mesons, from pion to Υ, lying on the leading Regge trajectories. The main parameters of the model, the universal string tension and the current quark masses, have been determined in [1] by

comparison with experimental meson masses lying on the ρ-type trajectories. So, for each other trajectory (without mixing), we have only one unknown short-range parameter. Experiment suggests that these parameters for the π-type and the b_1-type trajectories are equal. The short-range parameters do not strongly depend on the quark masses, and in some cases can be obtained from known parameters by a safe extrapolation in the quark masses.

As a result, the model predicts masses and other quantum numbers of many higher-spin mesons and some low-spin mesons. For instant, the model predicts a new $K^*(1^-)$ meson with mass 1910 Mev (without extrapolation) and the masses of $\eta_b(1S)(0^{-+})$ and $B_c(0^-)$ to be 9500 ± 30 MeV (bigger than the Υ-mass) and 6400 ± 30 MeV, respectively. The corresponding predictions of a potential quark model (PQM) [5] are 100 Mev lower. This number can characterize difference between many SQM and PQM predictions, so that further systematic experimental study of meson spectrum with accuracy capable to distinguish these predictions seems to be important for understanding confinement.

The SQM relativistic wave functions of composite mesons allow one to calculate the meson internal structure. The separate string and quark contributions into meson masses are obtained. The average spin projections of u- and $\bar{d}$-quark in polarized ρ^+, divided by the same projection of the total meson spin, are found to be 0.22 and 0.23, respectively, i.e., twice as small as the nonrelativistic quark model prediction 0.5.

The corresponding numbers for u- and $\bar{s}$-quark within K^{*+} are 0.22 and 0.42, respectively. This means that the flavour SU_3 is violated up to 80% for the spin structure in the relativistic model.

The results on the meson spin structure suggest a new approach to understanding the nucleon spin structure, obtained from polarized deep inelastic lepton-nucleon scattering and extrapolated to low Q^2 (the so called spin crises).

At the same time, the above numbers for the ρ spin structure are different from 0.17, the number corresponding to vanishing quark masses, so that one can hope that future polarization experiments will allow one to estimate the light-quark current masses from experiment.

The outline of the paper is the following. In Sec. 2, the string physics is described in the classical approximation. It follows, of course, from the string equations of Sec. 3, but can be described in familiar terms of the pointlike-particle mechanics, if only few basic properties of the string are taken from the equations. This description shows that the string, presumably realized by the gluon field inside mesons, is quite a new object from mechanical viewpoint. Sec. 2 also clarifies the origin and properties of the string functions, relevant to the quantum case.

In Sec. 3, the classical and quantum SQM is formulated and the meson wave functions are obtained. In going from classical SQM to the real one, we take into account quark spins, canonical quantization and nonstring, short-range, quark-antiquark interaction. All these effects are of the same order and all are necessary for consistency of the model. The quark spins are introduced at the classical level with the help of anticommuting variables obeying constraints [3]. We add a special term to the Lagrangian to ensure conservation of the spin constraints, which renders the total SQM Lagrangian supersymmetric. The canonical quantization implies finding out all the constraints between canonical variables, and using a first form method [4] to obtain the Poisson brackets of physical variables. As a result, the meson wave function satisfies two Dirac equations and a spectral condition, into which we introduce a nonstring, short-range, contribution. In general, the spectral condition may contain contributions dependent on the meson spin. Since we believe that the long-range contribution is given by the string term, then the additional short-range contribution can not increase with the meson spin. Experiment suggests that the short-range contribution does not depend on the spin or, for heavy quarkonia, has an additional, decreasing with the spin, term [1]. In this way we have phenomenological short-range parameters which depend on the type of the trajectory (i.e., on the space and charge-conjugation parity of the wave function) and on the quark masses. They obey the chiral symmetry (then the model obeys this symmetry) and, at present, are to be obtained from experiment.

In Sec. 4 and Appendix C, the spectral conditions for different meson wave functions are compared with the experimental meson spectrum, the model parameters are obtained and predictions of masses and other quantum numbers of new mesons are made. The results of SQM are compared with that of a potential quark model [5].

Knowing parameters of the model, we calculate in Sec. 5 the internal structure of mesons: the average values of string and quark energies, and projections of quark spins and orbital momentum for polarized mesons, as well as average total quark spin and orbital momentum squared, and spin-orbit correlation.

Sec. 6 contains conclusions. Some mathematical and phenomenological details are considered in Appendices A, B and C.

II. CLASSICAL STRING PHYSICS

The behaviour of a straight-line Nambu-Goto string, with or without point spinless quarks at the string ends, follows from the Lagrangian of the next Section. This behaviour can be described in terms of the point-particle relativistic

mechanics if we take from the Lagrangian three properties of the string. Let the string be in its rest frame, where the string is at rest as a whole, i.e., its 4-momentum is $(m, \mathbf{0})$. The specific string properties are the following.

I. The internal self-interaction string parameter a, called string tension, can be used as a "rest mass density" of the string.

II. The ends of an open string move with the velocity of light perpendicular to the string direction. The open string rotates in a plane around its center with an angular velocity

$$\omega = 2/d, \tag{1}$$

where d is the string length.

III. Point quarks with current masses m_i, $i = 1, 2$ at the ends of a rotating string do not move along the string. The string with quarks rotates in a plane. Its angular velocity and position of the rotation center are determined by equality of the centrifugal force and the string-tension force

$$\frac{m_i \omega^2 l_i}{\sqrt{1 - \omega^2 l_i^2}} = a\sqrt{1 - \omega^2 l_i^2}, \tag{2}$$

where l_i is the distance between the rotation center and the i-th quark. For zero-mass quarks, Eq. (2) is equivalent to Eq. (1). For heavy quarks $\omega l_i \ll 1$, and Eq. (2) reduces to

$$m_i \omega^2 l_i = a. \tag{3}$$

We see that the main peculiarity of the string is that it always rotates, and can not be stopped. If the quarks at the string ends are heavy and move slowly, so that their velocities $v_i \to 0$, then, from Eq. (3), $l_i = v_i^2 m_i / a \to 0$, and the string disappears. All points of the string can not be at rest, and the notion "rest mass density" is not applicable literally to the string. The property I above is a definition, following from the string Lagrangian. The string dynamics can not be reduced to the point-particle dynamics, although all other properties of the string can be obtained with its help.

To make illustrative estimates, we shall use the experimental value of a

$$a = 0.176\ GeV^2 \approx 1\ GeV/fm. \tag{4}$$

It is a huge "mass density" on the macroscopic scale.

Open string. From I and II, the energy of an open string, equal to its mass, is

$$E_0 = m = \int_{-d/2}^{d/2} \frac{a dx}{\sqrt{1 - \omega^2 x^2}} = \frac{1}{2}\pi a d, \tag{5}$$

or the length of the string is proportional to its mass

$$d = \frac{2}{\pi a} m. \tag{6}$$

The heavier a light-quark meson, the bigger it is. The lightest meson, the pion, would have $d \approx 0.1\ fm$.

¿From II, the angular velocity of the string is inversely proportional to its mass

$$\omega = \frac{\pi a}{m}. \tag{7}$$

For the pion it would be $\omega \approx 20\ fm^{-1} \approx 10^{24}\ Hz$. We shall see that the pion is not "the smallest top", but it is "the fastest one".

In the same way we can calculate the angular momentum of the string with respect to its rotation center

$$L = \int_{-d/2}^{d/2} x\ \frac{\omega x}{\sqrt{1 - \omega^2 x^2}}\ a dx = \frac{\pi a}{2\omega^2}, \tag{8}$$

or

$$L = \frac{1}{2\pi a} m^2. \tag{9}$$

Both sides of this equation are observable. This is a well-known linearly (with respect to m^2) rising Regge trajectory.

For the pion Eq. (9) yields $L \approx 0.02$, a comfortably small number.

Heavy-quark mesons. Let us introduce a meson mass excess

$$m_E = m - m_1 - m_2, \tag{10}$$

where m is the meson mass and m_1 and m_2 are the current quark masses, and let us consider

$$m_E/m_i \ll 1. \tag{11}$$

Then the motion is nonrelativistic, and the energies of the string and the quarks in Fig. 1b are

$$E_0 = ad, \tag{12}$$

$$E_i = m_i + \frac{1}{2}al_i. \tag{13}$$

The last equation follows from Eq. (3). Summing all these equations, we get

$$d = \frac{2}{3a}m_E. \tag{14}$$

The contribution of the string energy to the meson mass is small, but the contribution to m_E is not small,

$$E_0/m_E = 2/3 = 67\%, \tag{15}$$

and do not depend on the quark masses.

Eq. (14) reminds Eq. (6), where m is replaced by m_E and the slope is slightly bigger, to the extent that 3 is smaller than π.

We see that the string length in this case can be very small if m_E is small. Indeed, for the strange-quark current mass 0.22 GeV (Sec. 4), the diameter of the η-meson is smaller than that of the pion by 20%. The smallest particle is Υ, the b-quark mass being 4.71 GeV (Sec. 4). The Υ diameter is 0.02 fm, 1/5 that of the pion.

On the contrary, the behaviour of the angular velocity of the heavy-quark mesons is quite different from the open-string case. From Eqs. (3) and (14) it is easy to get

$$\omega = \frac{a}{\sqrt{\frac{2}{3}\mu m_E}}, \tag{16}$$

where μ is the reduced quark mass

$$\mu = m_1 m_2/(m_1 + m_2). \tag{17}$$

The angular velocity of Υ is 1/5 that of the pion.

The string angular momentum is negligible in this case and the total angular momentum is sum of the quark angular momenta

$$L = \sum m_i \omega l_i^2 = \frac{a^2}{\omega^3 \mu}, \tag{18}$$

or

$$L = \frac{1}{a}\left(\frac{2}{3}m_E\right)^{3/2}\mu^{1/2}. \tag{19}$$

This is also an observable Regge trajectory, nonlinear in this case, but determined by the same parameter a.

Asymmetric mesons. Let one quark, with mass m_1, be heavy, and the other one be very light, i.e.,

$$m_E/m_1 \ll 1, \quad m_2/m_E \ll 1, \tag{20}$$

where

$$m_E = m - m_1. \tag{21}$$

The string and quark energies are

$$E_0 = al_1 + \frac{1}{2}\pi a l_2, \tag{22}$$

$$E_1 = m_1 + \frac{1}{2}al_1, \tag{23}$$

where, to a first approximation, l_1 can be neglected, and we obtain

$$d = \frac{2}{\pi a}m_E, \tag{24}$$

$$E_0/m_E = 1, \tag{25}$$

$$\omega = \frac{\pi a}{2m_E}, \tag{26}$$

$$L = L_0 = \frac{\pi a}{4\omega^2}, \tag{27}$$

or

$$L = \frac{1}{\pi a}m_E^2. \tag{28}$$

The diameter (24) has the same slope as that for the open string, Eq. (6), the string gives the main contribution to the meson mass excess m_E and the Regge trajectory (28), as a function of m_E^2, has slope twice as big as that for the open string, Eq. (9), although the corrections to the first approximation, which can be easily worked out, are not negligible in practice.

General mesons. For arbitrary quark masses

$$m = E_0 + \sum E_i = a\int_{-l_1}^{l_2} \frac{dx}{\sqrt{1-\omega^2 x^2}} + \sum \frac{m_i}{\sqrt{1-\omega^2 l_i^2}}, \tag{29}$$

$$L = L_0 + \sum L_i = a\omega\int_{-l_1}^{l_2} \frac{x^2 dx}{\sqrt{1-\omega^2 x^2}} + \sum \frac{m_i l_i^2 \omega}{\sqrt{1-\omega^2 l_i^2}}, \tag{30}$$

where l_i is given by Eq. (2).

Introducing

$$l = 1/\omega, \tag{31}$$

$$l_i = \sqrt{l^2 + m_i^2/(4a^2)} - m_i/(2a), \tag{32}$$

$$G(l) = a\int_{-l_1}^{l_2} \sqrt{l^2 - x^2}\; dx + \sum m_i \sqrt{l^2 - l_i^2} \tag{33}$$

we can rewrite Eqs. (29) and (30) in the form

$$m = G_l(l) \equiv y \sum (\arctan t_i + t_i^{-1}), \tag{34}$$

$$L = K(l) \equiv \frac{1}{2a}(ym - \sum m_i^2 t_i), \tag{35}$$

where index l means derivative with respect to l, $y = al$, $t_i = (al_i/m_i)^{1/2}$ and

$$K(l) = lG_l(l) - G(l). \tag{36}$$

Eqs. (34) and (35) define a Regge trajectory as an implicit function

$$L = K(l(m)), \tag{37}$$

where $l(m)$ is a solution of Eq. (34).

If the string moves as a whole with a velocity $\mathbf{v}$, its rotation slows down: the angular velocity acquires a factor $\sqrt{1-\mathbf{v}^2}$. Its length, in general, is not conserved. The length oscillates between its minimal (rest-frame) value d, when the string is perpendicular to the velocity, and its maximal value $d/\sqrt{1-\mathbf{v}_{pl}^2}$, when the string is parallel to the projection of the velocity on the rotation plane $\mathbf{v}_{pl}$.

The classical description might be not only illustrative for $L \gg 1$. To make the model realistic, we must quantize it and take into account quark spins and nonstring short-range interactions. This will be done in the next Section.

III. QUANTUM STRING PHYSICS

We shall use Lorentz- and gauge-covariant variables. The straight-line string is a straight line in the 4-dimensional space-time

$$X(\tau, \sigma) = r(\tau) + f(\tau, \sigma)q(\tau), \tag{38}$$

where τ and σ are time-evolution and space-position parameters, respectively, r is a 4-vector of a point on the straight line, q is an affine 4-vector of its direction, f is a Lorentz scalar labelling points on the string, and $f_i = f(\tau, \sigma_i)$, $i = 1, 2$ correspond to the string ends. The covariant description introduces superfluous, from physical viewpoint, variables, therefore, the string action must be invariant with respect to three τ-dependent gauge transformations: shift of r along q, multiplication of q by a function of τ, and reparametrization of τ. The Lagrangian must be invariant with respect to the first two transformations and have a property $\mathcal{L}(c\dot{z}) = c\mathcal{L}(\dot{z})$, where $\dot{z}$ is every τ-derivative and c is a function of τ. There is only one string variable which is Poincaré- and gauge-invariant

$$l = \sqrt{\dot{r}_\perp^2}/b, \tag{39}$$

(not to consider higher τ-derivatives), where $\dot{r}_\perp^\mu$ is the string velocity, perpendicular to the rotation plane,

$$\dot{r}_\perp^\mu = (g^{\mu\nu} + n^\mu n^\nu - \dot{n}^\mu \dot{n}^\nu/\dot{n}^2)\dot{r}_\nu, \tag{40}$$

$$n = q/\sqrt{-q^2}, \tag{41}$$

and b is an angular velocity of the string with respect to the auxiliary time τ

$$b = \sqrt{-\dot{n}^2}. \tag{42}$$

b is gauge-dependent, but the condition

$$b \neq 0, \tag{43}$$

which we assume, is gauge-independent since τ is monotonous. Then there is a physically distinguished point on the string, the instantaneous rotation center, and we can label the points on the string, in a gauge-invariant way, with respect to this center

$$x = \sqrt{-q^2}\, f - \dot{r}\dot{n}/b^2. \tag{44}$$

The classical Lagrangian of a meson in SQM consists of three terms

$$\mathcal{L} = \mathcal{L}_{str} + \sum \mathcal{L}_i + \mathcal{L}_{ss}, \tag{45}$$

the first one being the Nambu-Goto Lagrangian for a straight-line string, Eqs. (38)-(44),

$$\mathcal{L}_{str} = -ab \int_{x_1}^{x_2} \sqrt{l^2 - x^2}\, dx, \tag{46}$$

where a is a string-tension parameter.

The second term in Eq. (45) is sum of the Lagrangians for point massive spinning quarks [BM] having velocities of the string ends

$$\mathcal{L}_i = -m_i \sqrt{\dot{X}_i^2} - \frac{i}{2} \xi_i^M \dot{\xi}_{iM} - i \left(\frac{\dot{X}_i \xi_i}{\sqrt{\dot{X}_i^2}} - \xi_i^5 \right) b\lambda_i, \tag{47}$$

where m_i is the quark current mass and ξ_i^M and λ_i are quark-spin variables ($M = \mu, 5$, and $g^{MN} = diag\{1, -1, -1, -1, -1\}$), which anticommute with each other (and commute with other variables, including spin variables of the other quark).

The Lagrangian (47) contains spin-independent part

$$\mathcal{L}_{i0} = -m_i \sqrt{\dot{X}_i^2}, \tag{48}$$

spin-velocity term showing that that the spin variables ξ_i^M are canonically self-cojugate, and spin-constraint term, proportional to a Lagrange multiplier λ_i. The spin constraints must be conserved. To ensure this conservation, we shall find out the third term in Eq. (45) (which restore a supersymmetry of the total Lagrangian). Toward this end, let us first consider the spin-independent part of the Lagrangian

$$\mathcal{L}_0 = \mathcal{L}_{str} + \sum \mathcal{L}_{i0}. \tag{49}$$

The quark velocity $\dot{X}_i$ must be perpendicular to the string direction. This is a property of the minimal surface formed by straight lines which follows from the Euler-Lagrange equations for the full string under the assumption Eq. (38). The proof of this property is given in Appendix A.

Introducing orthonormal vectors

$$v^0 = \dot{r}_\perp/(bl), \quad v^1 = \dot{n}/b, \tag{50}$$

we can write

$$\dot{X}_i = b(lv^0 + x_i v^1). \tag{51}$$

The extremum condition for $\mathcal{L}_0$ with respect to x_i yields

$$x_i = (-1)^i l_i, \tag{52}$$

(Eq. (32)), and the Lagrangian takes the form

$$\mathcal{L}_0 = -bG(l), \tag{53}$$

where $G(l)$ is given by Eq. (33).

Let us rewrite this Lagrangian in the phase-space. The momenta conjugate to the coordinates r and q

$$p = -\partial\mathcal{L}/\partial\dot{r}, \quad \pi = -\partial\mathcal{L}/\partial\dot{q}. \tag{54}$$

are equal to

$$p = G_l(l)v^0, \tag{55}$$

$$\pi = (-q^2)^{-1/2}((\dot{r}v^1/b)p + K(l)v^1), \tag{56}$$

where index l stands for derivative with respect to l and $K(l)$ is given by Eq. (36). The momentum p is conserved due to translation invariance. It is the total meson momentum, and the meson mass is

$$m=\sqrt{p^2}. \tag{57}$$

We shall use the notations

$$n^0=p/m, \quad \pi_p^\mu=(g^{\mu\nu}-n^{0\mu}n^{0\nu})\pi_\nu. \tag{58}$$

¿From Eqs. (55) and (56)

$$\pi_p=(-q^2)^{-1/2}Kv^1. \tag{59}$$

We see that the phase-space variables obey three constraints

$$pq=0, \quad \pi q=0, \tag{60}$$

$$m=G_l(l), \tag{61}$$

$$\sqrt{q^2\pi_p^2}=K(l). \tag{62}$$

The third constraint is given by two Eqs. (61) and (62): we must solve one of them (e.g., the first one) to find out l as a function of m, and put this solution into the second equation. The l.-h.s. of Eq. (62) with constraints Eqs. (60) is (orbital) angular momentum of our system

$$\sqrt{q^2\pi_p^2}=\sqrt{-L^2}, \tag{63}$$

$$L_\mu=\epsilon_{\mu\nu\rho\sigma}p^\nu L^{\rho\sigma}/2m, \quad L^{\mu\nu}=r^{[\mu}p^{\nu]}+q^{[\mu}\pi^{\nu]}. \tag{64}$$

The canonical Hamiltonian of a τ-reparametrization-invariant system is zero, and the Hamiltonian of our system is a linear combination of the constraint functions [6]. We can rewrite the Lagrangian (49), (53) in the form

$$\begin{aligned}\mathcal{L}_0= & -(1/2)(p\dot{r}-r\dot{p})-(1/2)(\pi\dot{q}-q\dot{\pi})- \\ & -c\left(\sqrt{-L^2}-K(l(m))\right)-c_1pq-c_2\pi q.\end{aligned} \tag{65}$$

where c, c_1 and c_2 are arbitrary ($c=-b$) and $l(m)$ is given by Eq. (61).

¿From Eq. (48), we get the quark momenta

$$p_i=m_i\dot{X}_i/\sqrt{\dot{X}_i^2}, \tag{66}$$

which, from Eqs. (51), (52), (55), and (59), are equal to

$$p_i=(ln^0+(-1)^il_in^1)m_i/\sqrt{l^2-l_i^2}, \tag{67}$$

where $l=l(m)$ is given by Eq. (61) and

$$n^1=\pi_p/\sqrt{-\pi_p^2}. \tag{68}$$

Now it is not difficult to introduce the quark spins in a consistent way. We add to the r.-h.s of Eq. (65) the spin-velocity term and the spin-constraint term expressed through the quark momenta (67), and, to ensure the spin-constraint conservation, we replace the orbital angular momentum L_μ by total angular momentum

$$J_\mu=\epsilon_{\mu\nu\rho\sigma}p^\nu M^{\rho\sigma}/2m, \tag{69}$$

$$M^{\mu\nu}=r^{[\mu}p^{\nu]}+q^{[\mu}\pi^{\nu]}-i\sum\xi_i^\mu\xi_i^\nu. \tag{70}$$

As a result, we obtain the SQM Lagrangian

$$\mathcal{L} = -(1/2)(p\dot{r} - r\dot{p}) - (1/2)(\pi\dot{q} - q\dot{\pi}) - (i/2)\sum \xi_i^M \dot{\xi}_{iM} -$$
$$- i\sum (p_i\xi_i - m_i\xi_i^5)\lambda_i - c\left(\sqrt{-J^2} - K(l(m))\right) - c_1 pq - c_2\pi q. \tag{71}$$

One can express this Lagrangian through the configuration space variables by means of the inverse Legendre transformation. We shall not use the configuration-space form of the Lagrangian. For the sake of compliteness, it is given in Appendix B with an outline of its derivation.

Besides quark-spin terms, a nonstring short-range interaction must enter into the meson Lagrangian. This interaction cannot be fully described at the classical level and will be taken into account in the quantum equations.

The first line of Eq. (71) corresponds to the first differential form of our system which determines the Poisson brackets of the canonical variables [4]. Namely, if we denote the variables by y_n and the first form by $(1/2)\omega_{mn}y_m\dot{y}_n$, then the Poisson brackets are $\{y_m,\ y_n\} = \omega^{mn}$, where ω^{mn} is inverse of ω_{mn}. For instance, from Eq. (71),

$$\{\xi^M\ ,\ \xi^N\} = ig^{MN}. \tag{72}$$

The other brackets have the usual canonical form. In particular, the total spin J^μ has zero Poisson brackets with Lorentz scalars, therefore, the spin constraint functions have zero brackets with the Hamiltonian and are conserved. This justifies the choice of $\mathcal{L}_{ss}$ in the Lagrangian. The gauge constraint functions are in involution with respect to the Poisson brackets, due to properties of the gauge transformations, and are also conserved.

The second line of Eq. (71) is minus Hamiltonian. We can exclude the constants c_1 and c_2 by choosing the gauge conditions

$$p\pi = 0, \quad \pi^2 = -1. \tag{73}$$

Their conservation yields $c_1 = c_2 = 0$, and we must solve Eqs. (73) together with the corresponding constraints (60). Introducing four orthonormal vectors $e_\alpha,\ \ \alpha = 0, 1, 2, 3$

$$e_0 = p/m, \quad e_\alpha e_\beta = g_{\alpha\beta}, \tag{74}$$

we can write the solution in the form $(a, b, c = 1, 2, 3)$

$$\pi = k^{(a)}e_a, \quad q = \epsilon_{abc}k^{(a)}L^{(b)}e_c, \quad L = L^{(a)}e_a. \tag{75}$$

We shall use space-vector notations for the set $\{k^{(a)}\}$ and similar sets

$$\{k^{(a)}\} = \vec{k}. \tag{76}$$

¿From Eqs. (73) and (64)

$$\vec{k}^2 = 1, \quad \vec{k}\vec{L} = 0. \tag{77}$$

Using expansions

$$J = J^{(a)}e_a, \quad \xi_i = \xi_i^{(\alpha)}e_\alpha\ ,\ \ \xi_i^{(5)} = \xi_i^5, \tag{78}$$

we can rewrite the Lagrangian (71) in the form (up to a total derivative)

$$\mathcal{L} = -p\dot{z} - [\vec{k}\times\vec{L}]\dot{\vec{k}} - \frac{i}{2}\sum \xi_{i(M)}\dot{\xi}_i^{(M)} -$$
$$- c(\sqrt{\vec{J}^2} - K) + \sum (p_i^{(\alpha)}\xi_{i(\alpha)} - m_i\xi_i^5)\lambda_i, \tag{79}$$

where the new string coordinate is

$$z^\mu = r^\mu + \frac{1}{2}\epsilon_{abc}e_a^\nu\frac{\partial e_{b\nu}}{\partial p_\mu}J^{(c)} + \frac{i}{m}\sum \xi_i^{(0)}\xi_i^{(a)}e_a^\mu, \tag{80}$$

and, in 4-vector notations,

$$p_i^{(\alpha)} = (l, \ \ (-1)^i l_i\vec{k})m_i/\sqrt{l^2 - l_i^2}. \tag{81}$$

The variables $\vec{k}$ and $\vec{L}$ are not independent, but using, e.g., spherical angles to solve Eqs. (77), one can easily obtain from the Lagrangian (79) the following nonzero Poisson brackets

$$\{p^\mu, z^\nu\} = g^{\mu\nu} \tag{82}$$

$$\{L^{(a)}, L^{(b)}\} = \epsilon_{abc} L^{(c)}, \quad \{L^{(a)}, k^{(b)}\} = \epsilon_{abc} k^{(c)} \tag{83}$$

$$\{\xi_i^{(M)}, \xi_i^{(N)}\} = i g^{MN}. \tag{84}$$

The Hamilton equations of motion can be easily solved. The solution for spinless quarks was described in Sec. 2.

The quantization of our system is now straightforward. We replace p, z, $\vec{L}$, $\vec{k}$ and $\xi_i^{(M)}$ by operators and their Poisson brackets by commutators or anticommutators for ξ's (multiplied by $-i$), e.g.,

$$[\xi_i^{(M)}, \xi_i^{(N)}]_+ = -g^{MN}, \quad [\xi_1^{(M)}, \xi_2^{(N)}]_- = 0. \tag{85}$$

Assuming the second quark to be an antiquark, we take the following solution of these equations

$$\xi_1^{(\mu)} = \frac{1}{\sqrt{2}} \gamma^5 \gamma^\mu \otimes I, \quad \xi_1^{(5)} = \frac{1}{\sqrt{2}} \gamma^5 \otimes I \tag{86}$$

$$\xi_2^{(\mu)} = \xi_1^{(\mu)c} = I \otimes \frac{1}{\sqrt{2}} \gamma^{5c} \gamma^{\mu c}, \quad \xi_2^{(5)} = \xi_1^{(5)c} = I \otimes \frac{1}{\sqrt{2}} \gamma^{5c}, \tag{87}$$

where $\gamma^c = C\gamma C^{-1}$ and C is the charge-conjugation matrix.

The constraint functions become operators annihilating the wave function. In the representation where p and $\vec{k}$ are diagonal the internal part of the wave function $\delta(p - p')\Psi_{\alpha\beta}(\vec{k})$ satisfies the equations

$$(\hat{p}_1 - m_1)\Psi = 0\ , \tag{88}$$

$$\Psi(\hat{p}_2 + m_2) = 0\ , \tag{89}$$

$$(\sqrt{\vec{J}^2} - K - \sum_{n=1}^{4} a_n P_n)\Psi = 0. \tag{90}$$

In the third equation a new term has been introduced to account for the short-range nonstring interaction. In this term, a_n can depend on J, and P_n are four independent operators commuting with the Dirac operators in the first and second equations (for fixed $J \neq 0$ there are four independent states of two particles with spin 1/2).

Since a_n describes a short-range interaction, it cannot increase with J. For the majority of mesons we can take a_n as a constant independent of J. Only heavy quarkonia demand more complicated a_n, containing a decreasing with J contribution [1].

The choice of P_n is connected with the choice of meson states at fixed J and will be discussed after solving Eqs. (88) and (89).

The solution of the Dirac equations (88) and (89) is a 4×4 matrix

$$\Psi = \frac{1}{\sqrt{(1 + b_1^2)(1 + b_2^2)}} \begin{pmatrix} -b_2 \chi \vec{\sigma}\vec{k} & \chi \\ b_1 b_2 \vec{\sigma}\vec{k} \chi \vec{\sigma}\vec{k} & -b_1 \vec{\sigma}\vec{k}\chi \end{pmatrix} \tag{91}$$

where

$$b_i = \frac{l_i}{l + \sqrt{m_i l_i / a}}, \quad b_i \to \begin{cases} 1, & m_i \to 0 \\ 0, & m_i \to \infty \end{cases} \tag{92}$$

and χ is an arbitrary normalized 2×2 matrix.

We shall take

$$\chi = \chi_{jMlS}, \tag{93}$$

which are eigenfunctions of $\vec{j}^2, j^{(3)}, \vec{L}^2$ and $\vec{s}^2$ with eigenvalues $j(j+1), M, l(l+1)$ and $S(S+1)$, respectively, where $\vec{j} = \vec{L} + \vec{s}$ and $\vec{s}$ is a 2-dimensional quark spin

$$\vec{s} = \frac{1}{2}\vec{\sigma} \otimes 1 + \frac{1}{2} 1 \otimes \vec{\sigma}^c, \quad \vec{\sigma}^c = \sigma_2 \vec{\sigma} \sigma_2 = -\vec{\sigma}^* \tag{94}$$

and 1 stands for the 2×2 unit matrix. These functions can be easily constructed with the help of Clebsch-Gordon coefficients.

The corresponding functions Ψ, denoted by Ψ_{jMlS}, are eigenfunctions of $\vec{J}^2$ and $J^{(3)}$ with eigenvalues $j(j+1)$ and M, respectively, and eigenfunctions of space and charge-conjugation parities

$$P\Psi_{jMlS} = -(-1)^l \Psi_{jMlS}, \quad \Psi^c_{jMlS} = (-1)^{l+S} \Psi_{jMlS}. \tag{95}$$

The parity transformations are defined by

$$P\Psi(\vec{k}) = \gamma^0 \Psi(-\vec{k}) \gamma^0, \quad \Psi^c(\vec{k}) = C\Psi^T(-\vec{k}) C^T, \tag{96}$$

where C is the charge-conjugation matrix. The Dirac equations (88), (89) are charge-conjugation-invariant for $m_1 = m_2$.

We shall assume that mesons in the states $\Psi_{jM,j-1,1}$ and $\Psi_{jM,j+1,1}$ do not mix.

This means that mesons with definite C or G-parity are described by the wave functions $\Psi_{jMlS} \equiv \Psi_n$, and the operators P_n in Eq.(90) are projection operators

$$P_n \Psi_m = \delta_{mn} \Psi_n. \tag{97}$$

The index n takes four (two) values for fixed $j \neq 0 (j = 0)$: $n = 0$ for $l = j$, $S = 0$; $n = 1$ for $l = j$, $S = 1$; and $n = \pm$ for $l = j \pm 1$, $S = 1$. Eq. (90) for these states takes the form

$$\sqrt{j(j+1)} = K + a_n \tag{98}$$

which is called the spectral condition. Here K is a function of m, m_1, m_2 and a, given by Eqs. (36), (33), and (32).

As mentioned before, a_n can be taken independent of j for all mesons except $c\bar{c}$- and $b\bar{b}$-mesons in the states with $n = -$, for which

$$a_- = A + \left(\frac{8m_1}{m(2j+1)^2} \right)^2 B \ , \tag{99}$$

where A and B do not depend on j.

For strange, charmed and bottom mesons the states Ψ_0 and Ψ_1 can mix for $j > 0$, so that the mesons are described by

$$\begin{aligned} \Phi_0 &= \cos\alpha \ \Psi_0 + \sin\alpha \ \Psi_1 \\ \Phi_1 &= -\sin\alpha \ \Psi_0 + \cos\alpha \ \Psi_1, \end{aligned} \tag{100}$$

and in the spectral condition (98) a_0 and a_1 are replaced by $a_0 - d$ and $a_1 + d$, respectively, where d is a mixing parameter

$$d = -b\tan\alpha = \frac{1}{2}(a_0 - a_1 \pm \sqrt{(a_0 - a_a)^2 + 4b^2}) \tag{101}$$

and upper (lower) sign corresponds to $a_0 - a_1 < 0 (> 0)$.

IV. MESON MASS SPECTRUM AND MODEL PARAMETERS

First comparison of the spectral condition (98) with experiment was made in Ref. [1,2]. Here we compare with more recent data available [7] and make predictions for mesons with spin up to 7.

The light quark current masses give very small contribution to the spectral condition and can not be determined from this condition. So we use the linear chiral SU_3 model relations [8]

$$m_u/m_d = 0.55, \quad m_s/m_d = 20.1 \tag{102}$$

to express them through the strange quark mass which is determined from comparison with experiment.

The best known meson Regge trajectories are described by the wave functions Ψ_- and have $P = C = (-1)^j$ and $j_{\min} = 1$. The parameters a_- or A in Eq. (99) depend very weakly on the quark masses: $a_-(d\bar{u}) = 0.88$, $a_-(c\bar{u}) = 0.90$, $A(c\bar{c}) = 0.90$, $a_-(b\bar{u}) = 0.77$, and $A(b\bar{b}) = 0.77$. This means that the short-range contributions for the strange quarks in these states are the same as for the light quarks: $a_-(s\bar{u}) = a_-(s\bar{s}) = a_-(d\bar{u})$, $a_-(c\bar{s}) = a_-(c\bar{u})$, and $a_-(b\bar{s}) = a_-(b\bar{u})$.

These trajectories allow one to determine the main parameters of the model [1,2]

$$a = 0.176 \pm 0.002 \ \text{GeV}^2$$

$$m_s = 224 \pm 7, \ \ m_c = 1440 \pm 10, \ \ m_b = 4715 \pm 20 \tag{103}$$

$$m_u = 6.2 \pm 0.2, \ \ m_d = 11.1 \pm 0.4$$

(masses in MeV), and the short-range parameters, Table IV in Appendix C.

A simpler procedure, when one drops the second term in Eq. (99), uses a_- independent of the quark masses and applies the minimum-χ^2-method [2], gives the same results for the main parameters (103) with good χ^2.

For the light- and strange-quark mesons the trajectories are practically linear.

For the light-heavy-quark mesons the trajectories are not linear but can be made practically linear by replacing the argument m^2 by $(m - m_h)^2$ where m_h is the heavy quark mass.

In the limit

$$\frac{2(m - m_h)}{\pi m_h} \ll 1, \quad \frac{\pi m_l}{2(m - m_h)} \ll 1, \tag{104}$$

where m_l is the light-quark mass, they must be linear with the slope twice as big as for the light-quark mesons. The trajectories are practically linear up to $j = 5$ with bigger effective slopes, but the first condition (104) for the limit slope is not fulfilled.

The trajectories for the heavy quarkonia are essentially nonlinear.

Table IV in Appendix C represents a detailed comparison of the model with experiment and contains predictions for new mesons and comparison with a potential model predictions [5]. The $B_J^*(5732)$-meson, found in ALEPH, DELPHI and OPAL experiments at CERN (see page 574 of Ref. [7] for the References) agrees with SQM much better than with PQM, Table IV.

The prediction for the $b\bar{c}, 1^-$-meson is made under the simplest assumption $a_-(b\bar{c}) = a_-(b\bar{u})$. The other assumptions: $a_-(b\bar{c}) = a_-(b\bar{b})$ or $a_-(c\bar{c})$ reduce its mass by a 100 MeV. The higher-spin $b\bar{c}$-mesons practically do not depend on this assumption.

The trajectories for Ψ_0 states, $C = -P = (-1)^j$ and $j_{\min} = 0$ are always nonlinear near $j = 0$ due to the square root in the spectral condition (98).

Only one parameter, the short-range contribution a_0, is unknown for each of these trajectories. It is determined from the mass of corresponding spin-0 meson.

We see that the nonlinear trajectory (98) with the universal slope describes quite well the three mesons π, b_1 and π_2.

The constant a_0 for the light-quark mesons is small. According to the linear chiral SU_3 model [8], it must be proportional to the light-quark masses m_l. This property does not contradict the spectral condition (98) where K is proportional to $m_l^{3/2}$.

There is not enough data to analyse Ψ_1 states, $P = C = -(-1)^j$ and $j_{\min} = 1$. So we assume that

$$a_1 = a_0 \tag{105}$$

for all mesons except $s\bar{u}$- and $s\bar{d}$-mesons for which mixing is important. Experiment confirms this assumption for known mesons composed by light, $s\bar{s}$- and $c\bar{c}$-quarks. If Eq. (105) is fulfilled also for the $b\bar{b}$-mesons then using the known mass of $\chi_{b1}(1P)$ we can estimate the mass of a pseudoscalar $b\bar{b}$-meson $\eta_b(1S), 0^{-+}$ to be 9.50 GeV which is bigger than the mass of $\Upsilon(1S)$.

Linearly extrapolating between $a_0(b\bar{u}) = -0.55$ and $a_0(b\bar{b}) = -0,091$, obtained from Eq. (105), we can find $a_0(b\bar{c}) = -0.41$ and estimate the mass of a pseudoscalar B_c-meson to be 6.40 GeV, which is 0.13 GeV higher than in the potential model [5].

We take into account mixing of Ψ_0 and Ψ_1 states only for strange mesons. The mixed states Φ_0 and Φ_1 (100) correspond to a mixing angle 36°. The detailed comparison with experiment and predictions for $\Psi_{0,1}$ ($\Phi_{0,1}$) states are given in Table V in Appendix C.

The behaviour of the trajectories for Ψ_+ states, $P = C = (-1)^j$ and $j_{\min} = 0$, is similar to that for Ψ_0.

The $X(1920), ?^{??}$-meson, found in GAMC and VES experiments at IHEP, Protvino, agrees quite well with SQM predictions and may be a 2^{++} trajectory partner of $a_0(980)$.

The strange mesons $K_0^*(1430), 0^+$ and $K^*(1680), 1^-$ are not described by the same wave function Ψ_+ (with different j). It seems probable that a new strange 1^- meson exists with mass 1900 MeV which is a partner of $K_0^*(1430)$, see Table VI in Appendix C. On the other hand, the $K^*(1680)$-mass, 1717 ± 17 MeV, is only half of its width, 322 ± 110 Mev, lower than the SQM value 1910 Mev.

We can tentatively conclude that the SQM descrides masses and other quantum numbers of about 2/3 of established mesons, the rest being daughter, gluball, or exotic states. The agreement with experiment for the former mesons is in general slightly better than that for the PQM. It seems important to continue systematic experimental study of meson mass spectrum where both models give different new predictions.

In conclusion of this Section, let us compare the description of heavy quarkonia in SQM and PQM. In the frame where the meson is at rest as a whole and the evolution parameter is the laboratory time, the SQM Hamiltonian is $m(J)$ where m is the solution of the spectral condition. Let us take Υ for definiteness and neglect the b-quark kinetic energy. Then the Hamiltonian is the sum of three terms

$$H = 2m_b + H_{string} + H_{short-range}$$

Considering the spectral condition without short-range contribution and using the current b-quark mass from Table 1C, it is not difficult to estimate the last term, so that (in MeV)

$$H = 9430 + 350 - 320$$

(Note that in Table 1 in the next Section the parameter E_0 comprises the string and the short-range contributions since it corresponds to x with account of the short range contribution in the spectral condition.)

We see that the string contribution is comparable with the short-range contribution. The confining role of the string for Υ is small: the string-tension energy is $a(l_1 + l_2)$=$2x^2/m_b$=20 MeV; but the kinetic energy of the string is not negligible.

The PQM Hamiltonian also contains three terms

$$H = 2M_b + H_{conf} + H_{short-range},$$

where M_b is the constituent quark mass which is bigger than the current mass. In Ref. [5] M_b=4977 MeV, $H_{conf} = c + br$= −70 MeV for Υ, so that in this case

$$H = 9950 - 70 - 420.$$

We see that the confinement potential is also small, and we need string to use current quarks instead of constituent ones.

V. INTERNAL STRUCTURE OF COMPOSITE MESONS

The model allows one to calculate quark velocities and energies and string energy in mesons at rest, Eqs. (61) and (81):

$$v_i = l_i/l, \quad E_i = l(am_i/l_i)^{1/2}, \quad E_0 = al \sum \arcsin v_i, \tag{106}$$

where l_i is given by Eq. (32) and l is a solution of Eq. (61), $l = x/a$ and x is given for each meson in Tables in Appendix C. The results for some mesons are collected in Table I.

TABLE I.Energy distribution inside mesons at rest. $v_i(E_i)$ is velocity in c (energy in MeV) of the i-th quark, E_0 is energy of the gluon string in MeV and $m_E = m - m_1 - m_2$.

Particle, quark content	v_1	v_2	E_1	E_2	E_0	$E_0/m, \%$	$E_0/m_E, \%$
$\rho^+, d\bar{u}$	0,98	0,99	53	39	679	88	90
$\pi^+, d\bar{u}$	0,88	0,93	23	16	99	72	82
$B^+, b\bar{u}$	0,07	0,99	4727	46	507	9.6	91
$J/\psi(1S), c\bar{c}$	0,22	0,22	1476	1476	146	4.7	67
$\Upsilon(1S), b\bar{b}$	0,05	0,05	4720	4720	22	0.2	67
$\chi_{b2}(1P), b\bar{b}$	0,18	0,18	4795	4795	324	3.3	67

We see that the light quarks are relativistic and give noticeable contributions to the meson masses. The main contribution to the mass "excess" of mesons $m_E = m - m_1 - m_2$ is given by the gluon string.

Let us consider spin structure of mesons, i.e., average values of internal angular momentum variables. The SQM allows one to calculate the spin structure of each meson on leading trajectories. The result depends on spin, parities and mass of the meson, string tension and current masses of quarks composing the meson. For instance, an average value of the third projection of the i-th-quark spin is given by

$$\overline{S_i^{(3)}} = (\Psi, S_i^{(3)}\Psi) = \int Sp\Psi^+ S_i^{(3)}\Psi d\vec{k}, \tag{107}$$

where

$$\vec{S}_1 = \frac{1}{2}\vec{\Sigma}\otimes I\ , \quad \vec{S}_2 = \frac{1}{2} I \otimes \vec{\Sigma}^c, \tag{108}$$

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & o \\ 0 & \vec{\sigma} \end{pmatrix}, \quad \vec{\Sigma}^c = \begin{pmatrix} \sigma_2\vec{\sigma}\sigma_2 & 0 \\ 0 & \sigma_2\vec{\sigma}\sigma_2 \end{pmatrix} = -\vec{\Sigma}^*. \tag{109}$$

Introduce the notations

$$c = 2(b_1^2 + b_2^2)N, \ \ c_1 = 4b_1^2 b_2^2 N, \ \ c_2 = 2(b_2^2 - b_1^2)N, \tag{110}$$

$$N = 1/((1+b_1^2)(1+b_2^2)) \tag{111}$$

where b_i is given by Eq. (92). In the nonrelativistic limit ($v_i = 0$, $m_1 + m_2 = m$) all c's vanish. In the ultrarelativistic limit ($v_i = 1$, $m_i = 0$) $c = c_1 = 1$ and $c_2 = 0$. Then, for a polarized meson with $J^{(3)} = M$, we obtain

$$\Psi_0 = \Psi_{jMj0}, \quad \overline{S_i^{(3)}} = 0, \tag{112}$$

$$\Psi_1 = \Psi_{jMj1}, \quad \overline{S_i^{(3)}} = \frac{M}{2j(j+1)}\ , \tag{113}$$

$$\Psi_- = \Psi_{jM,j-1,1}, \quad \overline{S_i^{(3)}} = \frac{M}{2}\left(\frac{1}{j} - \frac{1}{2j+1}(c + c_1 + (-1)^i c_2)\right)\ , \tag{114}$$

$$\Psi_+ = \Psi_{jM,j+1,1}, \quad \overline{S_i^{(3)}} = \frac{M}{2}\left(-\frac{1}{j+1} + \frac{1}{2j+1}(c + c_1 + (-1)^i c_2)\right)\ . \tag{115}$$

$$\overline{S^{(3)}} = \sum \overline{S_i^{(3)}}, \quad \overline{L^{(3)}} = M - \overline{S^{(3)}}. \tag{116}$$

In the same way, for squared quantities, we have

$$(\Psi_0, \vec{S}^2\Psi_0) = c, \ \ (\Psi_1, \vec{S}^2\Psi_1) = 2, \tag{117}$$

$$(\Psi_-, \vec{S}^2\Psi_-) = 2 - \frac{j}{2j+1}c, \tag{118}$$

$$(\Psi_+, \vec{S}^2\Psi_+) = 2 - \frac{j+1}{2j+1}c. \tag{119}$$

$$(\Psi_0, \vec{L}^2\Psi_0) = j(j+1) + c, \;\; (\Psi_1, \vec{L}^2\Psi_1) = j(j+1), \tag{120}$$

$$(\Psi_-, \vec{L}^2\Psi_-) = j\left(j - 1 + c + \frac{2j+2}{2j+1}c_1\right), \tag{121}$$

$$(\Psi_+, \vec{L}^2\Psi_+) = (j+1)\left(j + 2 - c - \frac{2j}{2j+1}c_1\right). \tag{122}$$

$$\overline{\vec{L}\vec{S}} = (1/2)\left(j(j+1) - \overline{\vec{L}^2} - \overline{\vec{S}^2}\right). \tag{123}$$

The spin structure of some mesons is represented in Tables II and III.

TABLE II. Spin structure of some mesons. Average values of internal angular momentum variables are shown for polarized mesons with $J^{(3)} = M$. nr— nonrelativistic limit , r — real case and ur — ultrarelativistic limit .

	$\overline{S_1^{(3)}}/M$			$\overline{S_2^{(2)}}/M$			$\overline{S^{(3)}}/M$			$\overline{L^{(3)}}/M$		
	nr	r	ur	nr	r	ur	nr	r	ur	nr	r	ur
$\rho^+, u\bar{d}$	1/2	0,22	1/6	1/2	0,23	1/6	1	0,45	1/3	0	0,55	2/3
$K^{*+}, u\bar{s}$		0,22			0,42			0,63			0,37	

TABLE III. Continuation of Table II. Average values do not depend on the meson polarization.

	$\overline{\vec{S}^2}$			$\overline{\vec{L}^2}$			$\overline{\vec{S}\vec{L}}$		
	nr	r	ur	nr	r	ur	nr	r	ur
π^+, ud	0	0,83	1	0	0,83	1	0	-0,83	-1
$K^+, u\bar{s}$		0,79			0,79			-0.79	
ρ^+, ud	2	1,68	5/3	0	1,87	7/3	0	-0,77	-1
$K^{*+}, u\bar{s}$		1,70			1,17			-0,43	

We see that the spin structure of light-quark mesons (114) is essentially different from the nonrelativistic case: the average quark spin projections are twice as small. The spin structure of ρ-meson in SQM is similar to the nucleon spin structure measured in experiment and different from the nonrelativistic quark model predictions.

The spin structure is also different from the ultrarelativistic case, when the light-quark current masses are neglected. Unlike the spectral condition, the spin structure is sensitive to the light-quark current masses. Measurement of the spin structure allows one to estimate the light-quark current masses from experiment.

We see also that the flavour SU_3 is badly broken in the spin structure of spinning mesons. The average value of the $\bar{s}$ spin projection in K^* is 80% bigger than the $\bar{d}$ spin projection in ρ.

VI. CONCLUSIONS

The gluon string in SQM can account for the quark confinement in mesons.

The string comprises two mechanisms of potential quark models (PQM): confinement potential and constituent quark masses.

Systematic experimental study of meson spectroscopy is important to check the SQM predictions in comparison with the PQM predictions.

Spin structure of light-quark vector mesons in SQM is different from the nonrelativistic quark model (NQM): for the average light-quark spin projections $\bar{S}_{SQM} \cong \frac{1}{2}\bar{S}_{NQM}$.

The flavour SU_3 is badly broken in the spin structure in SQM: for s and d quarks $\bar{S}_s \cong 2\bar{S}_d$.

Experimental study of the spin structure may eventually provide experimental estimation of the light-quark current masses.

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Appendix A: Lagrangian for a straight-line string with massive spinless quarks at the ends

This Lagrangian must give equations of motion which follow from the full-string Lagrangian with quarks at the ends, $i = 1$ or 2

$$\left(\partial L/\partial \dot{X}\right)^{\cdot} + \left(\partial L/\partial X'\right)' = 0, \tag{124}$$

$$(-1)^i \left((\partial L(\sigma_i)/\partial \dot{X})\dot{\sigma}_i - \partial L(\sigma_i)/\partial X'\right) + \left(\partial L_i/\partial \dot{X}_i\right)^{\cdot} = 0, \tag{125}$$

where $L(L_i)$ is the Nambu-Goto (the i-th-quark) Lagrangian, and dot (prime) stands for derivative with respect to τ (σ). For a straight-line string in the notations of Sec. 3

$$X(\tau,\sigma) = r(\tau) + (x(\tau,\sigma) + z(\tau))n(\tau), \tag{126}$$

$$z = \dot{r}v^1/b, \tag{127}$$

$$w = \dot{x} + \dot{z} - \dot{r}n, \tag{128}$$

we can rewrite Eq. (124) in the form

$$\left(x'(lv^0 + xv^1)/s\right)^{\cdot} - \left(w(lv^0 + xv^1)/s + bsn\right)' = 0, \tag{129}$$

$$s = \sqrt{l^2 - x^2}. \tag{130}$$

Using four orthonormal vectors v^0, v^1 (Eqs. (50)), n (Eq. (41)) and

$$v^2_\mu = \epsilon_{\mu\nu\rho\sigma} v^{0\nu} v^{3\rho} v^{1\sigma}, \quad v^3 = n, \tag{131}$$

$$v^a v^b = g^{ab}, \quad a,b = 0,1,2,3, \tag{132}$$

we can expand the l.-h.s. of Eq. (129) with respect to these vectors and get three equations (the fourth one, corresponding to the n-component, turns out to be an identity)

$$(x'l/s)^{\cdot} + \alpha x'x/s - (wl/s)' = 0, \tag{133}$$

$$(x'x/s)^{\cdot} + \alpha x'l/s - (wx/s)' = 0, \tag{134}$$

$$\beta l + \gamma x = 0, \tag{135}$$

where

$$\alpha = -\dot{v}^0 v^1, \quad \beta = -\dot{v}^0 v^2, \quad \gamma = -\dot{v}^1 v^2. \tag{136}$$

Since x is the only function which depends on σ, we get from Eq. (135)

$$\beta = \gamma = 0. \tag{137}$$

Eqs. (133) and (134) coincide

$$\dot{l}x - \alpha(l^2 - x^2) + (\dot{z} - \dot{r}n)l = 0 \tag{138}$$

and give

$$\dot{l} = \alpha = \dot{z} - \dot{r}n = 0. \tag{139}$$

So, Eq. (124) for the straight-line string is equivalent to

$$\dot{l} = 0, \quad \dot{v}^0 = 0, \quad \dot{v}^1 = -bn, \quad \dot{z} - \dot{r}n = 0. \tag{140}$$

Let us consider Eq. (125)

$$(-1)^i \left(\dot{x}_i(lv^0 + x_i v^1)/s + bs_i n\right) + m_i \left(\dot{X}_i/\sqrt{\dot{X}_i^2}\right)^{\cdot} = 0, \tag{141}$$

where dot means the total derivative with respect to τ,

$$s_i = \sqrt{l^2 - x_i^2}, \tag{142}$$

and

$$\dot{X}_i = b(lv^0 + x_i v^1) + (\dot{x}_i + \dot{z} - \dot{r}n)n. \tag{143}$$

Using Eqs. (140), we get

$$\dot{x}_i = 0, \tag{144}$$

$$(-1)^i a s_i - m_i x_i/s_i = 0. \tag{145}$$

Eq. (144) yields that the quarks can not move along the straight-line string. Eq. (145) coincides with Eqs. (52) and (32).

It is not difficult to check that the Lagrangian (49), (46), (48) and (51), used in Sec. 3, gives exactly the same equations of motion (140). (144) and (145).

Appendix B: The SQM Lagrangian in the configuration space

Neglecting a total τ-derivative, we can rewrite the SQM Lagrangian (71) in the form

$$\mathcal{L} = -p\dot{r} - \pi\dot{q} - (i/2)\sum \xi_i^M \dot{\xi}_{iM} - H, \tag{146}$$

$$H = c\left(J - K + i\sum F_{ia} c_i^a \lambda_i\right) + c_1 pq + c_2 \pi q, \tag{147}$$

where

$$c_i^a = n^a \xi_i, \quad c_i^5 = \xi_i^5, \tag{148}$$

$$n^0 = p/m, \quad n^1 = \pi_1/\sqrt{-\pi_1^2}, \quad n_\mu^2 = \epsilon_{\mu\nu\rho\sigma} n^{0\nu} n^{3\rho} n^{1\sigma}, \tag{149}$$

$$n^3 = q_p/\sqrt{-q_p^2}, \tag{150}$$

$$\pi_1^\mu = (g^{\mu\nu} - n^{0\mu}n^{0\nu} + n^{3\mu}n^{3\nu})\pi_\nu, \; ; \; q_p^\mu = (g^{\mu\nu} - n^{0\mu}n^{0\nu})q_\nu, \tag{151}$$

$$c_i^{ab} = c_i^a c_i^b, \quad c_{i,j}^{ab,ef} = c_i^{ab} c_j^{ef}, \tag{152}$$

$$J = \sqrt{-J^2} = L(1+t) + i\sum c_i^{13}, \quad t = \frac{1}{2L^2}\sum (c_{i,j}^{12,12} + c_{i,j}^{23,23}), \tag{153}$$

$$L = \sqrt{q_p^2 \pi_1^2}, \tag{154}$$

$$K = \bar{l}m - G(\bar{l}), \quad m = G_{\bar{l}}(\bar{l}) \tag{155}$$

$$F_{i0} = \bar{l}\sqrt{am_i/\bar{l}_i}\,, \quad F_{i1} = (-1)^i\sqrt{am_i\bar{l}_i}\,, \quad F_{i5} = -m_i. \tag{156}$$

Here $\bar{l}_i$ is given by Eq. (32) with substitution $\bar{l}$ for l where $\bar{l}$ is a solution of the second Eq. (155).

The velocity variables are determined by the inverse Legendre transformation

$$\dot{r} = -H_{(p)}, \quad \dot{q} = -H_{(\pi)}, \tag{157}$$

where index in brackets means the corresponding derivative. We can use constraints following from Eq. (147) after the differentiation. We get

$$\dot{r} = cl_0 n^0 + cy - c_1 q, \tag{158}$$

$$l_0 = \bar{l} + \delta_0, \quad \delta_0 = -i\sum F_{ia(m)} c_i^a \lambda_i, \tag{159}$$

$$y = -J_{(p)} - i\sum F_{ia} c_{i(p)}^a \lambda_i, \quad yn^0 = 0, \tag{160}$$

$$\dot{q} = c\alpha\pi_1 + c\sqrt{-q^2}\,\gamma n^2 - c_2 q, \tag{161}$$

$$\alpha = \sqrt{q_p^2\pi_1^2}\,(1-t). \tag{162}$$

$$\gamma = i\sum\left(\frac{1}{J}c_i^{13} + \frac{1}{L}F_{i1}c_i^2\lambda_i\right). \tag{163}$$

Eqs. (146), (147), (158) and(161) yield

$$\mathcal{L} = -c\left(G(\bar{l}) + \delta_0 m + 2Lt + i\sum (c_i^{13} + F_{ia}c_i^a\lambda_i)\right) - (i/2)\sum \xi_i^M \dot{\xi}_{iM}. \tag{164}$$

¿From Eq. (161), we get

$$c = b(1+\delta_b), \tag{165}$$

$$\delta_b = t - (1/2)\gamma^2, \tag{166}$$

$$v^1 = (1-(1/2)\gamma^2)n^1 + \gamma n^2. \tag{167}$$

Eqs. (158), (160) and (161) make it possible to find out l and v^0:

$$l = \sqrt{\dot{r}_\perp}\,/b = l_0(1-\delta_l), \tag{168}$$

$$\delta_l = -\delta_b + (1/2)\epsilon^2, \tag{169}$$

$$\epsilon = (b_2 - b_1\gamma)/l, \tag{170}$$

$$b_1 = \frac{1}{m}\sum\left[ic_i^{03} - \frac{1}{L}c_{i,j}^{02,12} + i(F_{i0}c_i^1 + F_{i1}c_i^0)\lambda_i\right], \tag{171}$$

$$b_2 = \frac{1}{m}\sum\left[\frac{1}{L}(c_{i,j}^{01,12} - c_{i,j}^{03,23}) + iF_{i0}c_i^2\lambda_i\right], \tag{172}$$

$$v^0 = (1 + (1/2)\epsilon^2)n^0 + \epsilon(-\gamma n^1 + n^2). \tag{173}$$

Since $v^3 = n^3$ on the constraints' surface, we can use Eqs. (167), (173) and (131) to get

$$v^2 = (1 + (1/2)\epsilon^2 - (1/2)\gamma^2)n^2 - \gamma n^1 + \epsilon n^0. \tag{174}$$

The next step is to express all functions of $\bar{l}$ as functions of l using Eqs. (159) and (169),

$$\bar{l} = l + l_1, \tag{175}$$

$$l_1 = -\delta_0 + l\delta_l, \tag{176}$$

and property of the Grassmann variables

$$l_1^4 = 0. \tag{177}$$

Finally, we must express c_i^a and their products through the velocity variables

$$u_i^a = v^a\xi_i, \quad a = 0, 1, 2, 3, \quad u_i^5 = \xi_i^5, \tag{178}$$

$$u_i^{ab} = u_i^a u_i^b, \quad u_{i,j}^{ab,ef} = u_i^{ab}u_j^{ef}, \tag{179}$$

where v^a are given by Eqs. (50), (41), (131), (167), (173), (174) and (175). Using properties of the Grassmann variables, we obtain the Lagrangian (45), (46), (47) and (51)

$$\mathcal{L} = -b\left(G(l) + i\sum F_{ia}u_i^a\lambda_i\right) - (i/2)\sum\xi_i^M\dot{\xi}_{iM} + \mathcal{L}_{ss}, \tag{180}$$

where

$$\mathcal{L}_{ss} = -b\left(A + i\sum B_i\lambda_i + C\lambda_1\lambda_2\right), \tag{181}$$

$$A = i\sum u_i^{13} - K^{-1}u_{1,2}^{12,12}, \tag{182}$$

$$B_i = \left(-\frac{1}{lG'K}F_{i0} + \frac{l}{K^2}F'_{i0}\right)u_{i,j}^{012,12} - \frac{i}{K}F_{i1}\left(u_{i,j}^{2,23} - \frac{2}{lG'K}u_{i,j}^{012,0123}\right), \tag{183}$$

$$\begin{aligned}
C = &-\frac{1}{G''}\sum_{a,b}F'_{ia}u_i^aF'_{jb}u_j^b + \left(\frac{1}{lG'}F_{i0}F_{j0} + \frac{1}{K}F_{i1}F_{j1}\right)u_{i,j}^{2,2} + \\
&+ \frac{i}{K}S\left[\left(\frac{1}{G''}F'_{i1} - \frac{l}{K}F_{i1}\right)F'_{j0} + \frac{1}{lG'}F_{i1}F_{j0}\right]u_{i,j}^{2,023} + \\
&+ \frac{i}{K}S\left[\left(\frac{1}{G''}F'_{i1} - \frac{l}{K}F_{i1}\right)F'_{j1} - \frac{1}{K}F_{i1}F_{j1}\right]u_{i,j}^{2,123} - \\
&- 2F_{i1}F_{j1}\left(\frac{1}{lG'K^2}u_{i,j}^{023,023,} + \frac{1}{K^3}u_{i,j}^{123,123}\right) - \\
&- \left[\frac{2}{lG'K^2}F_{i1}F_{j1} + \frac{1}{lG'K}\left(\frac{1}{G'} + \frac{l}{K}\right)SF_{i0}F'_{j0} + \frac{l}{G''K^2}SF'_{i0}F''_{j0} + \right. \\
&+ \left.\left(-\frac{2l^2}{K^3} + \frac{1}{G''K}\left(\frac{1}{K} - \frac{lG'''}{KG''} - \frac{2}{lG'}\right)\right)F'_{i0}F'_{j0}\right]u_{i,j}^{012,012}.
\end{aligned} \tag{184}$$

Here all the functions depend on l, prime stands for derivative with respect to l and

$$SX_{ij} = X_{ij} + X_{ji}. \tag{185}$$

The conserved spin constrains are

$$\sum_a F_{ia} u_i^a + B_i - iC\lambda_j = 0, \tag{186}$$

where $i = 1$ or 2 and $j \neq i$.

Appendix C: Comparison with experiment and potential quark model, parameters and predictions.

Comparison of SQM with experimental meson spectrum and a potential quark model, SQM parameters and predictions are collected in Tables IV, V and VI below for different meson trajectories called in correspondence with their lowest states.

q stands for quarks composing mesons;

j^{PC} means j^P for mesons not having C- or G-parity;

y (in GeV) is a main kinematical parameter of each meson, $y = al$, where l is a solution of Eq.(61) and a is the string tension. Knowing y, one can easily calculate all the other parameters of the meson wave function. In the classical approximation, a/y is angular velocity of the string;

m (in MeV) is SQM prediction for meson mass;

m_{EXP} (in MeV) is experimantal meson mass from Ref. [7] if no reference is indicated;

• indicates particles that appear in the Meson Summary Tables [7];

m_P (in MeV) is a potential quark model prediction from Ref. [5];

n is a meson name; and question marks stand for experimentally unknown j^{PC}.

TABLE IV. Vector trajectories (wave functions $\Psi_{jM,j-1,1}, P = C = (-1)^j, j_{\min} = 1$).

q	j^{PC}	y	m	m_{EXP}	m_P	n
$d\bar{u}$	1^{--}	0.2450	771	$\bullet 770.5 \pm 0.8$	770	$\rho(770)$
				$\bullet 781.94 \pm 0.12$	780	$\omega(782)$
	2^{++}	0.4196	1319	$\bullet 1318.1 \pm 0.6$	1310	$a_2(1320)$
				$\bullet 1275.0 \pm 1.2$		$f_2(1270)$
	3^{--}	0.5383	1692	$\bullet 1691 \pm 5$	1680	$\rho_3(1690)$
				$\bullet 1667 \pm 4$		$\omega_3(1670)$
	4^{++}	0.6346	1994	$\bullet 2060 \pm 20$[9]	2010	h/f_4 (2050)
				$\bullet 2010 \pm 20$[10]		$a_4(2040)$
	5^{--}	0.7179	2256	2330 ± 35[11]	2300	$\rho_5(2350)$
	6^{++}	0.7924	2490	2510 ± 30[12]		$r/f_6(2510)$
	7^{--}	0.8603	2703			
$s\bar{u}$	1^{-}	0.2600	893	$\bullet 891.66 \pm 0.26$	900	$K^*(892)^{\pm}$
				$\bullet 896.10 \pm 0.28$		$K^*(892)^0$
	2^{+}	0.4331	1418	$\bullet 1425.6 \pm 1.5$	1430	$K_2^*(1430)^{\pm}$
				$\bullet 1432.4 \pm 1.3$		$K_2^*(1430)^0$
	3^{-}	0.5509	1781	$\bullet 1776 \pm 7$	1790	$K_3^*(1780)$
	4^{+}	0.6465	2077	$\bullet 2045 \pm 9$	2110	$K_4^*(2045)$
	5^{-}	0.7293	2334	$2382 \pm 14 \pm 19$		$K_5^*(2380)$
$s\bar{s}$	1^{--}	0.2758	1013	$\bullet 1019.413 \pm 0.008$	1020	$\phi(1020)$
	2^{++}	0.4469	1516	$\bullet 1525 \pm 5$	1530	$f_2'(1525)$
	3^{--}	0.5636	1870	$\bullet 1854 \pm 7$	1900	$\phi_3(1850)$
	4^{++}	0.6586	2160		2200	
	5^{--}	0.7408	2413		2470	
	6^{++}	0.8145	2640			
$c\bar{u}$	1^{-}	0.3031	2008	$\bullet 2010.0 \pm 0.5$	2040	$D^*(2010)^{\pm}$
				$\bullet 2006.7 \pm 0.5$		$D^*(2007)^0$
	2^{+}	0.4996	2460	$\bullet 2458.9 \pm 2.0$	2500	$D_2^*(2460)^0$
				$\bullet 2459 \pm 4$		$D_2^*(2460)^{\pm}$
	3^{-}	0.6264	2777		2830	
	4^{+}	0.7269	3039		3110	
	5^{-}	0.8127	3269			
$c\bar{s}$	1^{-}	0.3244	2121	$\bullet 2112.4 \pm 0.7$	2130	$D_s^{*\pm}, ?^?$
	2^{+}	0.5163	2553	$\bullet 2573.5 \pm 1.7$	2590	$D_{sJ}(2573)^{\pm}, ?^?$
	3^{-}	0.6411	2861		2920	
	4^{+}	0.7405	3118		3190	
	5^{-}	0.8255	3344			
$c\bar{c}$	1^{--}	0.3309	3097	$\bullet 3096, 88 \pm 0.04$	3100	$J/\psi(1S)$
	2^{++}	0.6116	3557	$\bullet 3556.17 \pm 0.13$	3550	$\chi_{c2}(1P)$
	3^{--}	0.7412	3825		3850	
	4^{++}	0.8415	4050		4090	
	5^{--}	0.9267	4250			
$b\bar{u}$	1^{-}	0.3629	5327	$\bullet 5324.9 \pm 1.8$	5370	B^*
	2^{+}	0.5717	5716	5698 ± 12	5800	$B_J^*(5732), ?^?$
	3^{-}	0.7131	5994		6110	
	4^{+}	0.8262	6224		6360	
	5^{-}	0.9228	6426			
$b\bar{s}$	1^{-}	0.3875	5432	5416.3 ± 3.3	5450	B_s^*
	2^{+}	0.5920	5803		5880	
	3^{-}	0.7311	6073		6180	
	4^{+}	0.8427	6298		6430	
	5^{-}	0.9383	6497			
$b\bar{c}$	1^{-}	0.5169	6489		6340	
	2^{+}	0.7292	6780		6770	
	3^{-}	0.8681	7003		7040	
	4^{+}	0.9781	7195		7270	
	5^{-}	1.0717	7368			
	1^{--}	0.2274	9463	$\bullet 9460.37 \pm 0.21$	9460	$\Upsilon(1S)$

$b\bar{b}$	2^{++}	0.8850	9912	$\bullet 9913.2 \pm 0.6$	9900	$\chi_{b2}(1P)$
	3^{--}	1.0544	10106		10160	
	4^{++}	1.1791	10267		10360	
	5^{--}	1.2829	10411			

$$a = 0.176 \pm 0.002\ \mathrm{GeV}^2,$$
$$m_s = 224 \pm 7,\ m_c = 1440 \pm 10,\ m_b = 4715 \pm 20,$$
$$m_u = 6.2 \pm 0.2,\ m_d = 11.1 \pm 0.4,$$
$$a_-(d\bar{u}) = a_-(s\bar{u}) = a_-(s\bar{s}) = 0.88 \pm 0.01,$$
$$a_-(c\bar{u}) = a_-(c\bar{s}) = A(c\bar{c}) = 0.90,\ B(c\bar{c}) = 1.43,$$
$$a_-(b\bar{u}) = a_-(b\bar{s}) = a_-(b\bar{c}) = A(b\bar{b}) = 0.77,\ B(b\bar{b}) = 3.14$$

TABLE V. Pseudoscalar and pseudovector trajectories (wave functions Ψ_{jMj0}, $C=-P=(-1)^j$, $j_{\min}=0$ and Ψ_{jMj1}, $P=C=-(-1)^j$, $j_{\min}=1$, or mixed states Eqs. (100) for strange mesons).

q	j^{PC}	y	m	m_{EXP}	m_P	n
$d\bar{u}$	0^{-+}	0.04311	138	•139.56995 ± 0.00035	150	$\pi^\pm$
				•134.9764 ± 0.0006		π^0
	1^{+-}	0.4006	1259	•1229.5 ± 3.2	1220	$b_1(1235)$
				•1170 ± 20		$h_1(1170)$
	1^{++}			•1230 ± 40	1240	$a_1(1260)$
				•1281.9 ± 0.6		$f_1(1285)$
	2^{-+}	0.5258	1653	•1670 ± 20	1680	$\pi_2(1670)$
	2^{--}				1700	
	3^{+-}	0.6247	1963		2030	
	3^{++}				2050	
	4^{-+}	0.7093	2229		2330	
	4^{--}				2340	
	5^{++}	0.7847	2466			
$s\bar{s}$	0^{-+}	0.09231	548	•547.30 ± 0.12	520	η
	1^{+-}	0.4307	1468		1470	
	1^{++}			•1426.2 ± 1.2	1480	$f_1(1420)$
				1512 ± 4		$f_1(1510)$
	2^{-+}	0.5532	1838		1890	
	2^{--}				1910	
	3^{+-}	0.6503	2135		2220	
	3^{++}				2230	
	4^{-+}	0.7338	2391		2510	
	4^{--}				2520	
	5^{++}	0.8082	2621			
$c\bar{c}$	0^{-+}	0.2219	2980	•2979.8 ± 2.1	2970	$\eta_c(1S)$
	1^{+-}	0.6075	3548	3526.14 ± 0.24	3520	$\eta_c(1P), ?^{??}$
	1^{++}			•3510.53 ± 0.12	3510	$\chi_{c1}(1P)$
	$2^{-\pm}$	0.7385	3819		3840	
	3^{+-}	0.8395	4045		4090	
	3^{++}				4100	
	$4^{-\pm}$	0.9250	4246			
	0^{-+}	0.3361	9501		9400	
$b\bar{b}$	1^{+-}	0.8655	9892		9880	
	1^{++}			•9891.9 ± 0.7		$\chi_{b1}(1P)$
	$2^{-\pm}$	1.0357	10083		10150	
	$3^{+\mp}$	1.1633	10245		10350	
	$4^{-\pm}$	1.2690	10390			
$s\bar{u}$	0^-	0.1209	494	•493.677 ± 0.016	470	$K^\pm$
				•497.672 ± 0.031		K^0
	1^+	0.4390	1436	•1402 ± 7	1380	$K_1(1400)$
		0.3978	1310	•1273 ± 7	1340	$K_1(1270)$
	2^-	0.5576	1802	•1816 ± 13	1810	$K_2(1820)$
		0.5261	1704	•1773 ± 8	1780	$K_2(1770)$
	3^+	0.6528	2097		2150	
		0.6262	2014		2120	
	4^-	0.7351	2352		2440	
		0.7116	2279		2410	
	5^+	0.8087	2582			
		0.7875	2516			
$c\bar{u}$	0^-	0.2366	1869	•1869.3 ± 0.5	1880	$D^\pm$
				•1864.6 ± 0.5		D^0
	1^+	0.5228	2516	•2422.2 ± 1.8	2490	$D_1(2420)^0$
	2^-	0.6465	2828			
$c\bar{s}$	0^-	0.2491	1972	•1968.5 ± 0.6	1980	$D_s^\pm$
	1^+	0.5350	2598	•$2535.35 \pm 0.34 \pm 0.5$	2570	$D_{s1}(2536)^\pm$

	2^-	0.6576	2903			
$b\bar{u}$	0^-	0.3364	5279	$\bullet 5278.9 \pm 1.8$	5310	$B^\pm$
				$\bullet 5279.2 \pm 1.8$		B^0
	1^+	0.6153	5800			
	2^-	0.7495	6067			
$b\bar{s}$	0^-	0.3501	5368	$\bullet 5369.3 \pm 2.0$	5390	B_s^0
	1^+	0.6290	5873	5853 ± 15		$B_{sJ}(5850), ?^?$
	2^-	0.7624	6135			
$b\bar{c}$	0^-	0.4411	6403	$6400 \pm 390 \pm 130$	6270	B_c
	1^+	0.7516	6814			
	2^-	0.8876	7036			

$a_0(d\bar{u}) = a_1(d\bar{u}) = -0.016,\ a_0(s\bar{s}) = a_1(s\bar{s}) = -0.034,$
$a_0(c\bar{c}) = a_1(c\bar{c}) = -0.084,\ a_0(b\bar{b}) = a_1(b\bar{b}) = -0.091,$
$a_0(s\bar{u}) = -0.10,\ \ a_1(s\bar{u}) = 0,\ d(s\bar{u}) = 0.10$
$a_0(c\bar{u}) = a_1(c\bar{u}) = -0.30,\ a_0(c\bar{s}) = a_1(c\bar{s}) = -0.27,$
$a_0(b\bar{u}) = a_1(b\bar{u}) = -0.55,\ a_0(b\bar{s}) = a_1(b\bar{s}) = -0.51,$
$a_0(b\bar{c}) = -0.41$ (linear extrapolation between $a_0(b\bar{u})$ and $a_0(b\bar{b})$),
$d(c\bar{u}) = d(c\bar{s}) = d(b\bar{u}) = d(b\bar{s}) = d(b\bar{c}) = 0$

TABLE VI. Scalar trajectories (wave functions $\Psi_{jM,j+1,1}$, $P = C = (-1)^j$, $j_{\min} = 0$.)

q	j^{PC}	y	m	m_{EXP}	m_P	n
$d\bar{u}$	0^{++}	0.3143	988	$\bullet 983.4 \pm 0.9$	1090	$a_0(980)$
	1^{--}	0.5073	1594	$\bullet 1700 \pm 20$	1660	$\rho(1700)$
				$\bullet 1649 \pm 24$		$\omega(1600)$
	2^{++}	0.6110	1920	$1924 \pm 14[9,13]$	2050	$X(1920), ?^{??}$
	3^{--}	0.6979	2193		2370	
	4^{++}	0.7746	2434			
$s\bar{u}$	0^+	(I) 0.4358	(I) 1426	$\bullet 1429 \pm 6$	1240	$K_0^*(1430)$
		(II) 0.3493	(II)1162			
	1^-	(I) 0.5927	(I) 1910	$\bullet 1717 \pm 27$	1780	$K^*(1680)$
		(II) 0.5330	(II)1726			
	2^+	(I) 0.6846	(I) 2196		2150	
		(II) 0.6339	(II) 2038			
	3^-	(I) 0.7639	(I) 2442		2460	
		(II) 0.7189	(II) 2302			
	4^+	(I) 0.8352	(I) 2664			
		(II) 0.7943	(II) 2537			
$s\bar{s}$	0^{++}	0.2726	1004	$\bullet 980 \pm 10$	1360	$f_0(980)$
	1^{--}	0.4922	1653	$\bullet 1680 \pm 20$	1880	$\varphi(1680)$
	2^{++}	0.6018	1986	$\bullet 2011^{+60}_{-80}$	2440	$f_2(2010)$
	3^{--}	0.6918	2262		2540	
	4^{++}	0.7701	2505			
$c\bar{c}$	0^{++}	0.5357	3414	$\bullet 3417.3 \pm 2.8$	3440	$\chi_{c0}(1P)$
	1^{--}	0.7319	3805	$\bullet 3769.9 \pm 2.5$	3820	$\psi(3770)$
	2^{++}	0.8360	4037		4090	
	3^{--}	0.9224	4240			
	0^{++}	0.8340	9860	$\bullet 9859.8 \pm 1.3$	9850	$\chi_{b0}(1P)$
$b\bar{b}$	1^{--}	1.0663	10121		10140	
	2^{++}	1.1905	10282	$\bullet 10268.5 \pm 0.4$	10350	$\chi_{b2}(2P)$
	3^{--}	1.2930	10426			

$a_+(d\bar{u}) = -0.88$, (I)$a_+(s\bar{u}) = -1.59$, (II)$a_+(s\bar{u}) = -1.0,$
$a_+(s\bar{s}) = -0.52,\ \ a_+(c\bar{c}) = -1.06,\ a_+(b\bar{b}) = -1.35$

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